Precession of Spinning Tops

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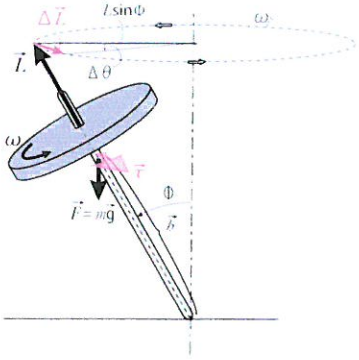


1. Objectives Measurement of revolution rates of spinning tops and their precession. Comparison with theory.

2. Theory The angular velocity of the precession ω_p is obtained as follows:

$$\omega_p = \frac{mgh}{I\,\omega} \qquad (1)$$

ω_p: The angular velocity of the precession [rad/s]
m : The mass of a top [kg]
g : Gravitational acceleration rate [m/s²]
h : The height from a support to the center of gravity [m]
I : The moment of inertia [kg·m²]
ω : Angular velocity of a top [rad/s]

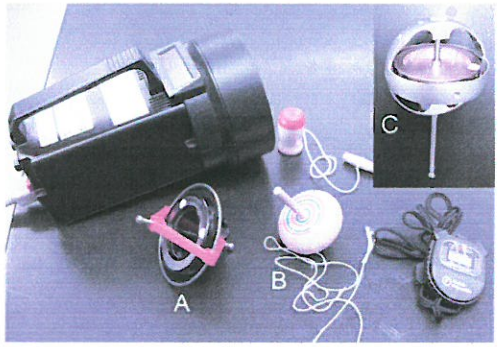
回転角速度ωと回転数fは次の関係がある。

$$\omega = 2\pi f$$

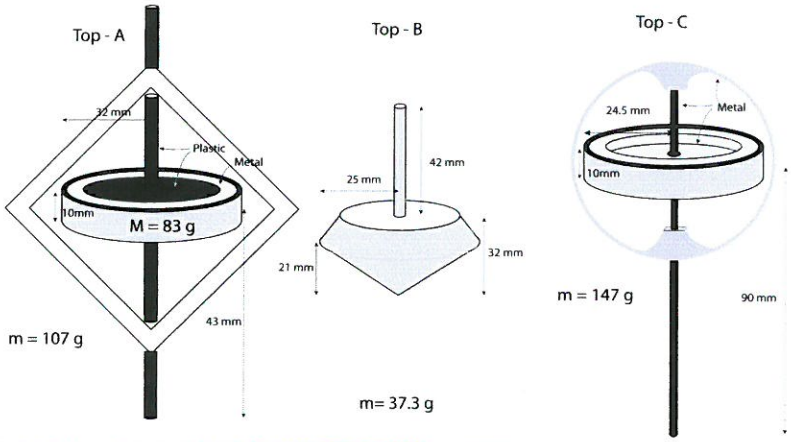
式(1)は次のように書き換えられる。

$$f_p = \frac{mgh}{4\pi^2 I\, f} \qquad (2)$$

f_p : Revolution rate of precession [Hz]
f : Revolution rate of a top [Hz]



3. Experiment
1） Three kinds of tops: A, B and C
2） Revolution of a top is measured with a stroboscope.



4. Results

		Top - A	Top - B	Top - C	
I		MR^2 M=83g	½ MR^2	½ mR^2	mR^2
	kg·m²	8.18×10^{-5}	1.17×10^{-5}	4.21×10^{-5}	8.42×10^{-5}
m	kg	107×10^{-3}	37.3×10^{-3}	147×10^{-3}	
h	m	43×10^{-3}	21×10^{-3}	90×10^{-3}	
f (obs)	rpm	1910	836	5251	
f (obs)	Hz	31.8	13.9	87.5	
fp (obs)	Hz	0.41	0.80	0.61	
fp (calc)	Hz	0.44	1.2	0.85	0.43

5. Discussion

There is ambiguity about the calculation of the moment of inertia. In Top-A, fp (obs) and fp (calc) agree within 7% on the model of a hoop ($I = MR^2$). In Top-C, fp (obs) shows the intermediate value between two values of fp (calc) calculation based on the two models, hoop ($I = MR^2$) and disk ($I = \frac{1}{2} MR^2$).