Fermi Questions

A Guide for Teachers, Students, and Event Supervisors

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Introduction to Fermi Questions

Fermi Questions are problems whose solutions are either too difficult to measure or whose answers are imprecise. Related calculations, so-called “back of the envelope calculations”, were an essential tool for scientists in the pre-computer era for they provided the means to keep track of exponents when using a slide rule. Performing (or setting up) **Fermi Question** calculations requires mathematical skills, logic, critical thinking, life experiences, and the ability to break down complex problems into smaller discrete, soluble parts. At some point in the solution process, the solver is expected to estimate a value which is critical to obtaining an answer. The methodology, involved in making up a **Fermi Question** as well as calculating an answer, is applicable not only to the fields of science and engineering but to those of finance and commerce; in short, to provide a solution to a question in any field requiring a numerical estimate (Ref. 1-5). In order to teach that methodology, I have written this manual, not only for teachers, but for prospective contestants as well as for Event Supervisors.

The **Fermi Questions** event in the Science Olympiad tests a team’s ability to estimate a solution to a problem by interpreting basic information, formulating a set of mathematical operations to provide an answer, and using mathematics to provide the answer to the question. Fundamental to the solution of these problems is a skill called *critical thinking* - essentially a method of attacking such problems in an orderly, logical way. This skill can be learned and it is the underlying basis for the event.

Over the years, I have observed that students grasp the essentials of problem solving by doing some. When they first tackle a **Fermi Question** problem, it may take several minutes for them to provide a solution. As they gain experience and become familiar with the methodology involved, the same problem may be solved in a fraction of a minute. Students reporting back to me after college or taking advanced degrees have stated that they were able to finish exams well before their contemporaries because of the skills learned during their preparation for the **Fermi Questions** event.

The basis of constructing a **Fermi Question** is a tribute to the person that the event is named after – the *Nobel Laureate* in Physics, Enrico Fermi. Fermi had a gift for solving complex problems: instead of trying to solve the problem all at once, he would break it up into small, solvable parts, and then combine those answers into a solution for the whole. For example, after watching the first atomic bomb explosion, he immediately calculated that the strength of the explosion was equivalent to the explosion of 10 kilotons of TNT. (Ref. 2) It took another three weeks for a panel of the Manhattan Project's best scientific brains to do an 'exact' calculation; their answer - 18 kilotons.

It is Fermi's methodology which should be followed, especially for Event Supervisors preparing a **Fermi Question** event. If a **Fermi Question** were posed in a non-competitive environment, the students should be expected to breakup the problem into smaller, discrete steps. In a competitive setting where time is at a premium, e.g., a ***Science Olympiad*** competition, the Event Supervisor should have the **Fermi Question¥** (the ultimate **Fermi Question** which will be denoted by the symbol **¥** in this brochure) as the last in a sequence of the questions as well as providing the discrete steps as questions preceding it. In this manner, the students see how the larger problem can be broken up into smaller, solvable steps and, in effect, they are able to get partial credit by solving each step.

Sample Illustration. An illustration at this point will provide an overview of the process. For those of you who might recall, in the opening portion of the film "***Finding Nemo***", the narrator states that there are 3.7 trillion ($3.7*10^{12}$) fish in the sea. In effect, the film has provided an answer to the **Fermi Question¥**: "How many fish are there in the oceans?" We will set up a path to calculate the answer to this question and, in doing so, provide a series of smaller, readily solvable problems. So, a solution path might involve these steps (working backwards from the desired answer):

3. Estimate the number of fish in the oceans (**Fermi Question¥**)
2. Estimate the volume of the oceans inhabited by most fish, m^3
1. Estimate the surface area of the Earth's oceans, km^2

Breaking up the problem for the students into steps and presenting them as discrete questions in the event allows the contestants to achieve partial credit for the overall solution. For the Event, I would present the questions in the numerical order given above. (note: I would not, as I understand some Event Supervisors have done, request the number of fish in Lake Superior without dividing the question into similar smaller, solvable parts.) Now, to solve each part in the order that they would be presented at the event (the correct order of magnitude is the FA or Fermi Answer):

1. Estimate the surface area of the Earth's oceans. Consider the Earth as a sphere of diameter $12.8*10^3$ km. The Earth's surface area is $\pi*D^2 = \pi*(12.8*10^3)^2 = 3*160*10^6$ km^2 $= 5*10^8$ km^2. The area of the oceans is about 70% of the Earth's total surface area. Therefore, the area of the Earth's oceans is $0.70*5*10^8$ km^2 $= 3.5*10^8$ km^2 FA = 8
2. Estimate the volume of the oceans habited by most fish, m^3. The volume equals the area of the oceans * depth. For this part, we have to estimate the depth below which there are few fish. Let's estimate this depth at 10 m. The habitable volume solution = $3.5*10^8$ km^2 * 10^6 m^2/km^2 * 10 m = $3.5*10^{15}$ m^3. FA = 15
3. Estimate the number of fish in the oceans (finally, we come to the **Fermi Question**[¥]). Another assumption must be made – that of the volume required by a fish to live, the cubic meters per fish. If we assume that the distance between fish is 20 m, then each fish has 10^3 m^3 to swim in (habitat). Dividing the total habitable volume by this value = $3.5*10^{15}$ $m^3/10^3$ m^3 per fish = $3*10^{12}$ fish. FA = 12

Discussion. The fact that we obtained close to the same answer as that given in the film is somewhat accidental. If we assumed slightly different values for the average habitable depth or the habitable volume for a fish, we might have obtained a different answer. These estimates are subjects for discussion by those trying to solve the problem. However, the estimates chosen for the illustration may be close to those assumed by the folks who did the calculation for the film. I searched the web to see if someone had presented the solution to the **Fermi Question**[¥] (the number of fish in the oceans) – I couldn't find any. Generally, when making up a **Fermi Question** event, search the web to see if similar problems or actual data exist that provide credibility to such a question (and the answer that is deemed correct).

The above illustration contains a number of features:

1. Once the **Fermi Question**[¥] is defined, the Event Supervisor needs to work backwards to identify the small, solvable steps. Generally, I use three steps as the limit to the number of questions dependent upon another answer.
2. The first question that I present is rather straightforward – virtually everyone gets the correct answer.
3. Expect the student to make appropriate estimates of critical values (in the illustration: the depth below which there are few fish; the volume required by a fish to live). The ability to arrive at reasonable estimates is one of the important attributes that must be learned by a prospective **Fermi Question** solver. And this attribute can be learned by solving these problems coupled with critical thinking.

4. Expect the solver to know formulae for the surface area of a sphere, circumference of a circle, etc. In the above illustration, the solver could assume that the Earth is a cube and still arrive at the correct answer.
5. The calculations show the need for the solver to be familiar with exponential notation. Otherwise, a lot of time will be required to write down all of the zeroes and keep track of them. Furthermore, since the answers are supposed to be the correct exponent, if the solvers don't know exponential notation, they're in the wrong event.

Fermi Question calculations can serve a variety of purposes, for example:

* provide estimates for a project before it is started thereby permitting a means to scope out the resources that are needed to accomplish same. For example, when you ask a wedding consultant to plan the affair, they often ask the question, "How many people will attend?" Your approximate answer will allow them to better plan the event. If your answer is 10 people, the consultant might say that the event could be held within a few days. All it might require is a couple of phone calls to invite the guests, set a time with a Justice of the Peace, call up a restaurant and make a reservation, etc. On the other hand, if the guest list is 100, then more time will be needed to print up invitations, mail and receive replies, rent a hall, line up suppliers of flowers and food, and musicians, etc., etc.

* estimate the feasibility of an opportunity. For example, can the community that you live in financially support another fast food emporium? You can arrive at an estimate by considering the local population, how many times a week they might go out to eat, and if the current businesses can satisfy the demand. If so, then the chances of making a go of your new eatery may be very slim and you should consider other opportunities.

* determine if an answer that you have obtained makes sense. In my work as a scientist at DuPont, I have sometimes found that strange analytical results or plant operational failures can be explained by using the methodology developed in solving Fermi Questions. More often than not, the assumptions that I need to make, to arrive at the measured result, lead to an understanding of the cause of the problem. Then, a simple experiment is generally called for to prove the point.

* provide the basis for a discussion. One subject of a Fermi Question that I have asked involves the geometric population growth of organisms. The requested solvable, smaller step questions might be:

- What is the volume of the Earth, cm^3?
- If the organism doubles every 6 minutes, how many will there be after 1 day?
- If the organism measures $1*10^{-4}$ cm by $2*10^{-4}$ cm by $5*10^{-4}$ cm, what is the ratio of their total volume to the Earth's at the end of 1 day?

The answer to the last question is larger than the Earth's volume. When a student once asked "why doesn't this happen?", that question lead to an interesting discussion. I pointed out that the **Fermi Question** was directed towards growth. What it didn't consider was that there was another competition – that of death – which would limit the organism's population. We then discussed some of those limiting factors: the normal lifetime of the organism might be shorter than one day; the organisms would run out of food; other organisms would eat them; their climate conditions might inhibit reproduction or kill them; and, as you might have expected, several other possibilities were presented.

I had been supervising Fermi Questions as a state event in Delaware for over twenty five years. It has been especially rewarding to me to watch how well the students (generally, teams of two) collaborate to solve the problems. Knowing how much effort I expend in making the exam (30-60 hours), I am gratified to watch these budding scientists expend their mental energies in kind. For that reason, I try to make the questions fun, a learning experience, and relevant to their quest for knowledge. Because learning to solve problems of varying orders of magnitude is such an important scientific and business attribute, I have provided copies of the **Fermi Questions** event to a half dozen university professors who then administered it to their students (both graduate and undergraduate) as a learning/teaching exercise.

Considerations in making up a Fermi Questions event:

* Math (straight) – where the answer can be calculated using a calculator or computer but, since such aids are not allowed in the competition, it forces the student to consider other routes to provide a reasonable answer

* How answers from one problem relate to other problems – as with many facets of life, an answer to one problem leads to many other choices and provides the stepping stone to solutions of more complex problems.

* Having solutions to problems relate to 'real life', for example, a problem might ask for an estimate of the amount of gasoline used by passenger cars in the U.S., how an increase in gas mileage of cars would relate to a decrease in green-house gas production, and how the amount of water produced by same relates to other items such as rainfall or filling of swimming pools.

* Behind each problem set that I create is the tacit assumption that the contestants have a reasonable knowledge of mathematics, specifically, the use and operations of exponential notation. The lack of math skills is not too apparent when the answers to the problems are in the range 0.001 to 1000 (**Fermi Question** notation –3 to +3). But when I ask the students to calculate the number of iron atoms on the head of a pin, the inability to handle exponents readily shows (there are approximately $3*10^{13}$ iron atoms – **Fermi Question** answer +13; see

Example xiv. below). I can't count the number of times that I've seen students cover the scrap paper (that I distribute for them to use in their deliberations) with zeroes. For that reason, it is imperative to stress the use of exponential notation (which also serves as the basis for the metric system). Not only does the use of exponential notation make calculations faster, but it also helps avoid problems with writing, transcribing, and counting the correct number of zeroes. So much so, that in some branches of science there are specially named units that have very large (or very small) numbers associated with them, such as, one Angstrom = 10^{-8} cm, one Light Year = $5.9*10^{12}$ miles, Avogadro's Number = $6.023*10^{23}$. If there are any doubters about the utility of using exponential notation, use a stopwatch to record the amount of time it takes them to write 6 followed by 23 zeroes versus writing $6*10^{23}$ or 6 E 23.

* As I noted above, an important component of the event is logical, critical thinking. Reading and understanding the problem is one important component; the other important component is to develop a plan to provide the answer in the requested units.

* And finally, time is a critical parameter. The ability to think and calculate rapidly can be learned – the keywords are, in the immortal words of a Hall-of-Fame football coach, "practice, practice, and more practice". I have watched students (when I was a coach) significantly lower the times required to solve these problems.

Typically, the first time that a team tries to solve a problem, they try to be too exact. For example, if the **Fermi Question** is "how many toothpicks are equivalent to the perimeter of Colorado?", they might discuss the length of a toothpick ("is it 2.0, 2.25, 2.45 inches?”); then they try to estimate the perimeter of the state; and finally, they calculate a value. Any time there is a discussion, time is lost. Since the answer to any question is the correct order of magnitude, an error of a factor of two or three will probably still yield the correct exponent (the **Fermi Question** answer). Hence, they should pick a reasonable value and work up their answer. The time that they save will be needed to solve other problems.

Why this event? Numbers (when you think about it) are a measure of our surroundings and life.

Here are a few examples of **Fermi Questions**:

- How many air molecules are in this room (where I was presenting this lecture)?
- How many pounds of CO_2 and H_2O does the U.S. population expel in a year?

- How many tons of food are consumed in Chicago during the course of a day?
- How many people are involved in delivering and preparing that food?
- How many gallons of paint do you need to paint the walls of your school?
- How many baseballs are used during the course of the Major League season?
- How many pizzas were eaten last year in the U.S.?

Always remember to have the current Rules for the event

Currently, the scoring for the event is:
5 points for the correct exponent
3 points for the correct exponent ± 1
1 point for the correct exponent ± 2

The answer to a Fermi Question is the correct exponent of 10 (if an answer is $5*10^n$, round the answer up to the next power of 10; I try to manage the problems so that answers are not $5.0*10^n$). Generally, if a team averages 3 points per problem and there are 30 problems, the 90 points that they will have achieved will garner them a medal. Calculators, computers, or any other device, including crib sheets, lists of constants, formulae, etc., are not permitted. All the contestants need are pencils (with erasers) and a good night's sleep - I supply scratch paper (to simulate the 'back-of-envelopes'). Positive exponential values are the default; negative exponents **MUST** have the - (**minus**) sign as part of the answer.

Considerations involved when learning to solve Fermi Questions.

The FA notation used below is short for **F**ermi **A**nswer (the order of magnitude).

1. **Exponents are short-hand notation** (knowledge of which makes it easier and faster to solve the problems).

What is the population of New York City? Answer $8{,}000{,}000 = 8*10^6 \sim 10^7$ FA **7**

What is the distance, in miles, from the Earth to the Sun? Answer $100{,}000{,}000 = 10^8$ FA **8**

2. **Properties of exponents**.

$500 = 5*10^2$; 5 is the coefficient, 10 is the base, 2 is the exponent

When multiplying, add exponents of the same base

$200 * 4000 = 2*10^2 * 2^2*10^3 = 2^3*10^5 = 8*10^5 \sim 10^6$ FA **6**

When dividing, subtract exponents of the same base

$200 \div 800 = 2*10^2 \div 2^3*10^2 = 2^{-2}*10^0 = 2^{-2}*1 \sim 10^{-1}$ FA **-1**

Notes: the minus sign must be included as part of the answer and $10^0 = 1$

$10^{20} = (10^4)^5 = (10^2)^{10}$

$2^{10} \sim 10^3$

3. **Round off values BEFORE doing a calculation**. Rounding off a value makes it much easier and faster to do the problems. **Why? Because the Fermi Answer is the correct order of magnitude** which means that there is a large range of values corresponding to the correct answer. For example, the Fermi Answer for the distance, in miles, from the Earth to the Sun is 8 (shown above in 1.) but the range of values giving the same answer is $5*10^7$ to $4.99*10^8$!! For **Fermi Question** calculations, I recommend using 1 significant figure (2 at most) and rounded off values. In this context, for ease and speed of calculation, I suggest using the values below which are somewhat different from the exact values:

Item	**Exact Value**	**Suggested Value (for faster calculation)**
1 day	24 hours	25 hours
1 mile	5280 feet	5000 feet
1 yard	0.9144 meter	1 meter
1 foot	30.48 cm	30 cm
1 pound	453.6 g	500 g
1 hour	3600 seconds	4000 seconds

4. **Always keep the units as part of working a problem**. In some instances, keeping track of the units will lead to the correct answer. I am particularly sensitive to the use of units since I have degrees in both engineering (British units are used, pounds, feet, BTU, etc.) and chemistry (metric units, grams, meters, calories, etc.) **AND** the U.S. uses both of these systems. As an example, most U.S. cooks know what a $^1/_4$ pound of butter looks like - it is a stick about 1 inch x 1 inch x 5 inches. But ask them what 100 g of butter looks like and they may throw up their hands in defeat. The answer is that the ‘metric’ stick is almost the same size as the U.S. stick since 100 g is close to $^1/_4$ pound. Sometimes the units get left off solutions to real problems with tragic, unforeseen results. For the **Fermi Questions** event, I indicate the units of the problems.

5. **What subject matter is covered**? Practically every subject is fair game! If the subject in question has numbers associated with it, this invites a **Fermi Question** to be asked. In the past, I have given questions on math, chemistry, physics, biology, geology, geography, economics, swimming, basketball, running, census, food, waste generation, etc. (see the section on “**Fermi Question** Problem Sets”)

Practice Examples. These can be done by the students as practice; have them show all work and what assumptions they made in solving the problems. (F?s = **Fermi Question** solution). Note that units are used in practice – whatever shorthand symbols the students use is acceptable (in the **Fermi Questions Event**, I specify the unit of the answer).

i. How many seconds are there in a year?

Exact solution: 60 sec/min * 60 min/hr * 24 hr/day * 365 day/year = $3.15*10^7$ s/y FA 7

F?s: 4000 s/h * 25 h/d * 400 d/y = $4*10^3 * 2.5*10^1 * 4*10^2 = 4*2.5*4*10^6 = 4*10^7$ FA 7

Note that both FA answers are the same. Budding Fermi Question experts should remember that there are $3*10^7$ seconds in a year (s/y) - this will probably save them time in future calculations.

ii. How many miles are there in a light-year?

Exact solution: 186,000 m/s * $3.15*10^7$ s/y = $1.86*3.15*10^{12} = 5.9*10^{12}$ m/y FA 13

F?s: $2*10^5$ m/s * $3*10^7$ s/y = $6*10^{12}$ m/y

This quantity is a basic unit used in astronomy. As noted in problem i., knowing that there are $3*10^7$ seconds in a year has shortened the work considerably.

iii. How many kilometers are there in a light-year?

F?s: $6*10^{12}$ mi/y * 1.6 km/mi = $10*10^{12}$ km/y = 10^{13} FA 13

iv. For the average American woman, how many times will her heart beat during her lifetime?

Assumptions: 1 heartbeat per second (hb/s), lifetime of 80 years

F?s: $3*10^7$ s/y * 1 hb/s * 80 y/lifetime = $2.4*10^9$ FA 9

v. How many heartbeats are there in a year for the entire world's population?

Assumptions: 1 heartbeat per second, $6*10^9$ people

F?s = $3*10^7$ s/y * 1hb/s * $6*10^9 = 18*10^{16} = 1.8*10^{17}$ FA 17

vi. How many pounds of rice were consumed in the U.S. in the year 2001?

Assumptions: 20 pounds of rice eaten per year by a person (20 #/p), $3*10^8$ people in the U.S.

F?s: 20 #/p * $3*10^8$ p = $6*10^9$ # ~ 10^{10} FA 10

This answer was checked using data from the U.S. Dept. of Agriculture; $5.2*10^9$ lbs. If the students assume 2-10 or 200-1000 #/p, they would still get 3 points.

vii. What is the density of butter in g/cc?

Assumptions: 1 pound of butter is a package 2 inch x 2 inch x 5 inches.

F?s: Volume = 2 in*2.5 cm/in * 2 in*2.5 cm/in * 5 in*2.5 cm/in = 5*5*12 = 300 cm^3

Density = Mass/Volume = 500 g / 300 cm^3 = 1.5 $g/cm^3 \sim 1 = 10^0$ FA 0

viii. What fraction of a mile is a cm?

F?s: 1 cm / (5000 f/mi * 30 cm/f) = $1 / 5*3*10^4 = 1 / 1.5*10^5 \sim 10^{-5}$ FA -5

ix. What is the area, in sq. miles, of the original 48 states of the U.S.?

Assumption: the U.S. is shaped like a rectangle; 3000 miles wide x 1000 miles

F?s: $3*10^3$ mi* $1*10^3$ mi = $3*10^6$ $mi^2 \sim 10^6$ FA 6

Note: when areas are requested, it is much easier to use a rectangle.

x. What is the area of the U.S. (prob. ix.) in cm^2?

F?s: $3*10^6$ mi^2 * 1.6^2 km^2/mi^2 * $(10^3)^2$ m^2/km^2 * $(10^2)^2$ cm^2/m^2

= $3*2.5 * 10^{(6+6+4)} = 7.5 * 10^{16} \sim 10^{17}$ $cm^2 \sim 10^{17}$ FA 17

xi. What is the area of Lake Superior in sq. miles?

Assumption: the lake is shaped like a rectangle; 300 miles wide x 100 miles

F?s: $3*10^2$ mi * $1*10^2$ mi = $3*10^4$ $mi^2 \sim 10^4$ FA 4

xii. Estimate the volume of Lake Superior in cubic kilometers.

Volume = Area * Depth; Assumption: Average depth is 200 m

F?s: $3*10^4$ mi^2 * 1.6^2 km^2/mi^2 * 200 m * 1 km/1000 m

= $3*2.5*2 * 10^{(4+2-3)} = 15 * 10^3 = 1.5*10^4$ km^3 FA 4

xiii. How many cubic kilometers of rain fall of the U.S. (48 original states) in one year?

Assume an average rainfall of 10 inches.

F?s: $3*10^6$ mi^2 * 1.6^2 km^2/mi^2 * 10 in * 2.5 cm/in * 1 m/10^2 cm * 1 km/10^3 m

= $3*2.5*2.5 * 10^{(6+1-2-3)} = 20 * 10^2 = 2 * 10^3$ km^3 FA 3

Note that Lake Superior has about 7 times the total volume of rainfall: the Great Lakes have about half of the Earth's fresh water.

xiv. How many iron atoms are on the head of a pin?

Assumptions: the diameter of the head is 1 mm; diameter of an iron atom is 2.5 Angstroms

Area of the head of a pin: $^1/_2 * \pi * D^2 = 1.5 * (1 \text{ mm} * 1 \text{ cm}/10 \text{ mm})^2 = 1.5 * 10^{-2}$ cm^2

Assume that the head of a pin is half a sphere

Area covered by an iron atom: $^1/_4 * \pi * D^2 = 0.75 * (2.5*10^{-8})^2$ $cm^2 = 5 * 10^{-16}$ cm^2

F?s: $1.5*10^{-2}$ cm^2 / $5*10^{-16}$ cm^2 = $0.3*10^{14}$ = $3 * 10^{13}$ FA 13

Note: this problem can also be solved using the rectangle approach.

Assumption: the head is a cube with a side of 1 mm; an iron atom is 2.5 Angstroms on a side

Area of the head of a pin: $^1/_2*6*S^2 = 3 * (1 \text{ mm} * 1 \text{ cm}/10 \text{ mm})^2 = 3 * 10^{-2}$ cm^2

Area covered by an iron atom: $S^2 = (2.5*10^{-8})^2$ cm^2 = $6.25 * 10^{-16}$ cm^2

F?s: $3*10^{-2}$ cm^2 / $6.25*10^{-16}$ cm^2 = $0.4*10^{14}$ = $4 * 10^{13}$ FA 13

References

1. http://en.wikipedia.org/wiki/Fermi_problem

2. ***My Observations During the Explosion at Trinity on July 16, 1945.*** **E. Fermi http://www2.vo.lu/homepages/geko/atom/report.htm**

3. The University of Maryland Physics Education Group maintains a collection of Fermi problems. **http://www.physics.umd.edu/perg/fermi/fermi.htm**

4. Natalie Angier, **The New York Times**, Tuesday, March 31, 2009, page D2, “The Biggest of Puzzles Brought Down to Size”.

5. John A. Adams and Lawrence Weinstein, “Guesstimation: Solving the World’s Problems on the Back of a Cocktail Napkin”, Princeton University Press (2008).

The following section contains some of the **Fermi Questions** for events that I ran and provides a general idea of the kinds of questions that I ask. When I first started running the **Fermi Questions** Event in Delaware 25^+ years ago, I employed a similar methodology that I use today. However, as the students' sophistication (and speed) in answering those questions improved (*i.e.*, the average scores increased over the years), I needed to add more complexity.

I had intended to include solutions to the problems but, it soon became apparent that it was taking me too long to type everything. Rather than wait any longer to issue this Guide, I left the solutions for the readers to work out. However, many of the answers to the problems can be found using various search engines on the internet. As I noted above, I generally check the web to make sure that the questions (and answers) are realistic.

2002 Invitational

1 How many inch-worms are equivalent in length to the circumference of the Earth at the equator?

2 How many sheets of paper, laid flat, will it take to reach the moon from the Earth?

3 Under favorable conditions, the bacterium SO (short for ***Scientificus Olympiadum***) will reproduce in 6 minutes. Assuming they don't die, how many SO bacteria can claim the same ancestor after 1 day?

4 The SO bacterium is rod-like with dimensions of $4x10^{-4}$ cm, 10^{-4} cm, and $0.5x10^{-4}$ cm. What is the weight, in grams, of an SO bacterium?

5 How many SO bacteria would be able to be packed into a ping pong ball?

6 The SO bacterium is special because it has a tiny brain which is about the same percentage of its total weight as an adult human's. Estimate the weight, in grams, of the brain of an SO bacterium.

7 The SO bacterium needs a living space of about $30x10^{-4}$ cm in all directions to thrive. How many of these thriving bacteria might be found in a gram of pond water?

8 What is the volume of the earth, in cm^3?

9 If the SO bacteria don't die, how many hours will it take for two to reproduce and make a volume equal to that of the earth?

10 How many drops of water are needed to fill a 50 meter swimming pool?

11 How many hours would it take the fastest person, running non-stop, to go from Anchorage to New York?

12 What fraction of volume does one water molecule occupy in a cup of water?

13 How many grams of steam would be needed to melt a cubic foot of ice?

14 How many times could a basketball player bounce a ball during her teenage years if she didn't stop to eat, sleep, etc.?

15 What mass, in grams, would a piece of spaghetti have if it could circle Colorado?

16 How many electrons pass through a lighted 60 watt bulb in a day?

17 What is the mass, in kgrams, of the moon?

18 How many different five card poker hands can you make from a 52 card deck?

19 How many toothpicks could be made from a telephone pole?

20 How many molecules are in a grain of beach sand?

21 Since the invention of the automobile, how many people have died in the United States as a result of car accidents?

22 If $1 was deposited in a savings account on July 4, 1776 at a guaranteed rate of 5% and no money was withdrawn, how many dollars are in the account today?

23 How many seconds would it take for the light from an explosion on Jupiter to reach the Earth?

24 How many seconds would it take the sound from a volcano's eruption to travel around the Earth?

25 Estimate the mass, in grams, of the fans in the stadium attending tomorrow's St. Louis Rams-Philadelphia Eagles football game.

26 What is the total height, in km, of all of the world's people?

27 What fraction of the sun's energy output is intercepted by the earth?

28 Silas Marner saved his pennies until they covered the 48 contiguous states of the United States. How many pennies did he have?

29 How many new telephone numbers became available when the three digit area code system was installed?

30 During the time interval of a blink, how many km has light traveled in a vacuum?

2002 National (Trial Event)

1 What is the surface area of the Earth in square miles?

2 What is the volume of the Earth's oceans in cubic miles?

3 For one day, what is the volume in cubic miles of the average rainfall on the Earth?

4 What is the volume of the Moon in cubic meters?

5 If all of the Earth's oceans were transported to the Moon, what would be the average depth, in meters, of the lunar ocean?

6 What fraction of the Moon's volume is represented by this new lunar ocean?

7 What fraction of the Earth's land mass is represented by Colorado?

8 If all of the people in the United States move to Colorado, what would be the population density, people per square mile, of that state?

9 If the Earth's average daily rainfall fell on Colorado, how many inches of rain precipitated?

10 How many 50-meter swimming pools could the water from Problem 9 fill?

11 Speedy Jones is the only deaf, Olympic sprinter. He uses the flash of the Starter's pistol to start a race. Speedy and 7 others won their 100-meter heats in 10.00 seconds. In the semi-final heat featuring these 8 racers, the Starter mistakenly fired his pistol at the finish line of the race. By how many millimeters did Speedy win his heat?

12 Assume that the average American is 10% overweight. How many pounds of fat does the U.S. population carry?

13 What is the volume, in cubic meters, of the fat in Problem 12?

14 If a gram of fat delivers the same energy content as a gram of gasoline, how many gallons of gasoline are equivalent to the fat of Problem 12?

15 For the United States, how many cars would have their fuel needs met for one year by the gasoline of Problem 14?

16 If all of the cars in the U.S. got at least 25 miles per gallon, how many gallons of gas would be saved this year?

17 How many tons of CO_2 would not be produced via the fuel saving of problem 16?

18 What is the mass, in kgm, of the Earth's atmosphere?

19 What fraction of the CO_2 in the Earth's atmosphere is represented by the CO_2 that was not released in problem 17?

20 How many hours elapse from one full moon to the next?

21 How far, in km, does light travel during the time elapsed in Problem 20?

22 Relative to the sun, what is the Earth's orbital velocity in miles per hour?

23 How many kilometers would the Earth have travelled during the elapsed time of Problem 20?

24 How many pounds of onions were eaten in the U.S. in 2000?

25 How many adult humans are needed to be a mass equivalent to the onions of Problem 24?

26 How many days would it take a person, working continuously, to peel the onions of Problem 24?

27 Under ideal conditions, the bacterium *eNSO* can reproduce in 10 minutes. How many *eNSO* can claim the same ancestor after 24 hours?

28 *eNSO* is rodlike with a diameter of 1 μm and a length of 5 μm. What is the volume, in cubic meters, of the *eNSO* of Problem 27?

29 What is the ratio of the volume of *eNSO* of problem 28 to the volume of the Moon?

30 If *eNSO* eats only onions as its food source, how many minutes elapse before it consumes an amount equal to the onion mass of Problem 24?

2004 – Regional

Blood accounts for 7% of the mass of a person. Erythrocytes or Red Blood Cells, RBCs, make up 45% (volume) of an adult male's blood and 42% of an adult female's. An RBC is in the shape of a disk, about 7.5 μm diameter by 2 μm thick (1μm = 10^{-6} m). RBCs live approximately $^{1}/_{3}$ of a year. Each RBC contains 2.8×10^{8} hemoglobin molecules that can each transport 4 oxygen molecules.

1 What is the volume of RBCs, in cc, in the blood of an adult male?

2 How many RBCs equal one cc in volume?

3 How many RBCs does an adult male human have?

4 If placed in a square pattern, what is the area, in m^2, that the RBCs of problem 3 would cover?

5 How high a column, in cm, would the RBCs of problem 3 form if stacked like a bunch of poker chips?

6 If fully oxygenated, how many oxygen molecules could be carried by the RBCs of problem 3?

7 For a typical male, how many RBCs die each day?

8 In a minute, all of a person's blood passes through the heart. During a day, how many times does a RBC pass through the heart?

9 In a male, how many oxygen molecules are transported by the blood during a day?

10 How many cubic centimeters of oxygen (STP) are transported during a year in an adult male?

11 How many gallons of blood does a person's heart pump in a lifetime?

12 What is the mass, in grams, of the RBCs in a male?

13 During a male's lifetime, how many pounds of RBCs are produced?

14 How many RBCs are created per second in an adult male?

15 How many more RBCs does an adult male have than an adult woman?

16 What is the volume, cc, of blood in a healthy new-born baby?

17 How many units (500 cc) of blood are donated each day in the United States?

18 How many people are afflicted with sickle cell anemia in the United States?

19 In the United States, how many people have type O blood?

20 If I donate a pint of blood, how many days will it take for my red cell count to return to normal?

21 For the entire United States populace, how many liters of blood are pumped each day?

22 Assume that all of the oxygen transported in problem 9 is converted to liquid water. What is the volume of that water in cc?

23 What fraction of an adult male's weight is represented by the water in problem 22?

24 What is the approximate total length, km, of blood vessels in an adult male?

25 How many adults donate blood each year in the United States?

26 What is the volume, cc, of a 25 yard long 6 lane (1 lane is 6 feet wide) swimming pool that is filled with water to a depth of 5 feet?

27 How many tons of oxygen are transported in the blood of the United State's populace each day?

28 What is the volume, cc, of blood donated each year in the United States?

29 How many swimming pools (prob. 26) can be filled by the blood in prob. 28?

30 What is the mass, grams, of RBCs in one swimming pool of prob. 29?

2004 National

Water, water, everywhere, nor any drop to drink.

1. What is the volume (cubic miles) of the earth?
2. What is the average depth (cm) of the earth's oceans?
3. What is the area (square miles) covered by the earth's oceans?
4. What is the volume (cubic meters) of the earth's oceans?
5. What fraction of the earth's volume is provided by the oceans?
6. How many tons of oxygen atoms are contained in a cubic mile of ocean?
7. How many water molecules are present in the earth's oceans?
8. How many water molecules are in a typical adult human male?
9. How many molecules of water are in a cup of water?
10. How many oxygen atoms in a cup of water once were part of the first human male?
11. How many drops of water are needed to fill a swimming pool (25x12x2 meters)?
12. How many grams of steam are needed to melt a cubic foot of ice?
13. What is the mass (grams) of a water molecule?
14. What is the pressure (atmospheres) at the bottom of the Mindinao trench in the Pacific Ocean?
15. How long (meters) is the shoreline of the North American continent?
16. What fraction of the earth's water is equal to the water in an adult male human?
17. If sea water is 3.5% of salt by weight, how many tons of salt are in a cubic mile of ocean?
18. If the oceans dried up, estimate the thickness (cm) of the resulting layer of salt uniformly covering the earth.
19. 38 pounds of gold are contained in a cubic mile of sea water. How many pounds of gold are contained in the world's oceans?
20. What is the weight fraction of gold in sea water?

21. If it were to rain an average of 1 cm per hour over the entire surface of the earth, how many cubic miles of water would have rained during the course of one day?

22. What is the fraction of the ocean's volume represented by the rainfall in problem 21?

23. If the contiguous 48 states were to get an average of 30 inches of rain per year, how many grams of water is that?

24. If it rained at the rate of a $^1/_4$ inch per hour, how many days would it take to fill the swimming pool of Problem 11?

25. The volume of ice in Antarctica is estimated to be 26 million cubic kilometers. If all of the ice in Antarctica melted, how high (feet) would the oceans rise?

26. The Gulf Stream is about 50 miles wide and 1500 feet deep and flows past Miami at 5 mph. How many tons of water in the Gulf Stream pass by Miami in a minute?

27. Estimate the mass, kg, of the earth's atmosphere.

28. If the average water vapor content of the atmosphere is 1% by weight, how many cubic miles of water is this equal to?

29. What fraction of the ocean's volume is the water of Problem 28 equal to?

30. How thick, meters, a water layer would be formed if all of the water in Problem 28 were to condense simultaneously all over the earth?

2005 National

Titanium dioxide, TiO_2, is the ubiquitous white pigment used in items such as photographic paper, paint, and plastics. TiO_2 has a density of 4.2 g/cc and the pigment particles are cubes with sides of 0.25µm. A paint formulation contains 42% (wt.) of TiO_2 pigment, 12% resin (density 1.2 g/cc) and the remainder is water. A gallon of this paint can coat 400 ft^2 of walls.

1. Estimate the consumption of pigmentary TiO_2 in the United States in 2004.
2. What is the volume, cc, of the TiO_2 in a gallon of the paint formulation (above)?
3. What is the mass, grams, of a gallon of the paint?
4. What is the mass, grams, of a TiO_2 pigment particle?
5. What is the surface area, cm^2, of a TiO_2 pigment particle?
6. What is the total surface area, m^2, of the pigment particles in a gallon of the paint?
7. If the paint is used to coat 400 ft^2 of walls, how thick, cm, is the paint film?
8. What is the average distance, m, between pigment particles in a paint film?
9. How many molecules are in a TiO_2 pigment particle?
10. If all of the TiO_2 pigment particles in the gallon of paint were place side by side as in a chain, how long, km, is the chain?

WaterWorks. The Earth's oceans cover an area of $3.6*10^8$ km^2 and their average depth is $4*10^3$m. The density of salt water is 1.027 g/cc.

11. What is the total mass, kgm, of the oceans?
12. How many water molecules are in the oceans?
13. If all of the oceans were frozen into a single ice cube, how high, km, would it be?
14. What is the volume, cc, of a water molecule?
15. What is the total mass, tons, of the components in the oceans beside water?
16. If the Earth's oceans covered a sphere the diameter of the moon, how deep, m, would the new ocean be?

17. What is the average pressure, atm, at the bottom of the Earth's oceans?

18. If a 6 foot diameter pipe were built to contain all the ocean water, how many miles long would the pipe have to be?

19. What is the ratio of the length of pipe (prob. 18) to the distance from the Earth to the Sun?

20. If the oceans were frozen into 1 inch ice cubes, how many cubes would there be?

21. If the ice cubes in prob. 20 were placed side by side to form a single line, how long, km, would the line be?

22. How many years would it take for light to travel the length of pipe in problem 18?

Rice Grains. A grain of rice is a rectangular prism, 2 mm x 2 mm x 7 mm long. 100 grains mass 1.7g. A cup of uncooked rice masses 100g and, when boiled with 2 cups of water, makes 3 cups of cooked rice. A rice grain contains 15% water and the rest can be written as $C_6H_{10}O_5$.

23. What is the density, g/cc, of an uncooked grain of rice?

24. How many grains of rice are in a 5 lbs bag?

25. For a cup of uncooked rice, what fraction of the cup is air?

26. 5 lbs of rice yields how many cc of cooked rice?

27. By cooking, how many times does rice expand in volume?

28. If 5 lbs of rice are cooked in ocean water (see **WaterWorks**), how many grams of salts are in the cooked rice?

29. 5 lbs of rice, if completely oxidized, yields how many grams of water?

30. A cup of cooked rice contains how many grams of the element carbon?

2006 National

Some Facts. Gold, atomic weight 197 and atomic number 79, has a nuclear radius of $7.3x10^{-15}$ m and an atomic radius of $1.3x10^{-10}$ m. A proton and neutron are of equal mass and 2000 times more massive than an electron. The Sun has a diameter of $1.4x10^{6}$ km and a mass of $2x10^{30}$ kg.

1. What is the mass, g, of a gold atom?
2. What is the density, g/cc, of the gold atom nucleus?
3. What is the mass, g, of the Earth?
4. Assuming that it has the same density as a gold nucleus, what is the diameter, cm, of a neutron star that has the same mass as the Earth?
5. If a gold nucleus was expanded to a diameter of 1 m, how far away, m, is the electron cloud?
6. What is the volume fraction of a gold atom occupied by matter?
7. Assuming that it has the same density as a gold nucleus, what is the diameter, cm, of an electron?
8. What is the volume fraction of the Solar System (from the Sun to Earth) occupied by the Sun, and the planets?
9. What is the density, g/cc, of the Sun?
10. If the Sun were to collapse into a neutron star, what would be its diameter, cm?
11. Given the same size reduction as in Problem 10, how far, in cm, is the Earth from the Sun?
12. During the course of a calendar year, how far, km, through space has the moon traveled?
13. How many seconds does it take for light to travel from the Sun to the Earth?
14. How many days would it take a sound wave to travel the same distance at sea level on Earth?

15. A breakfast cereal, similar to Cheerios, is composed of small oat cylinders, 1 cm dia., 4 mm high with a 4 mm diameter hole bored through. Estimate the mass, g, of one cylinder.
16. How many oat cylinders are in a cereal box containing 425 g?
17. Two of the dimensions of the cereal box are 30 and 20 cm. Estimate the width, cm, of the box.
18. The cereal is poured to fill an 8 oz. bowl. Milk is then added to completely fill the bowl. How many cc of milk were added?
19. A lawn has blades of grass that are rectangular in cross-section; 2 mm wide by 0.1 mm thick. There are 10 blades of grass in each 2 by 2 cm area of lawn. The owner mows the grass on his 1 acre lawn to a height of 7cm. For a freshly mowed lawn, estimate the mass, g, of the grass above ground.
20. The grass on the lawn grows 10 cm in height during the week between mowings. Estimate the mass, g, of the grass clippings collected each week.
21. The owner collects and dries the clippings. In the course of a six month growing season, what is the total mass, g, of the dried clippings?

The year is 2030. A limited access highway (cars only) was built from New Jack City to Ballmore. The road is 250 km long, 12 lanes wide (4 traffic lanes going north and 4 south) with pull-out lanes outside lanes 1 & 4. The road bed is 30 cm thick. A 30 cm concrete barrier separates the north/south roads and 30 m wide grass strips are on the outer portions of the highway. The speed limit is 100 km/hr. E-Z pass toll booths (cars speed through) are situated at the ends and exits of the highway. A driver is expected to maintain a safe distance of 100 m between his/her car and the vehicle in front.

22. What is the volume, m^3, of the road bed of the highway?
23. On Sunday of Thanksgiving weekend, the highway is at maximum capacity with cars safely going at the speed limit. How many cars are on the highway?
24. For the period when the highway is operating at maximum capacity, how many cars exit from the highway per hour?

25. What is the total area, m^2, covered by the highway and the grass outer strips?
26. If all of the land that the highway was built upon contained houses with 1 acre lots, how many houses had to be torn down to build the highway?
27. If cars get 20 km/liter gas mileage in 2030, how many liters per hour would be consumed during the period of use at maximum capacity?
28. On the average, during the day the highway is 40% occupied while, at night, it is 10 % full. In a year, how many cars use the highway?
29. What is the total gasoline consumption, gallons, for cars using the highway in a year?
30. On the grass strip going north, a high speed train (250 km/hr) track was built; shortly thereafter, a track was installed on the south grass strip. Assuming that each train was composed of 12 cars with a capacity of 100 people per car. How often, in minutes, would the interval between trains have to be in order to transport the same amount of people that would use the highway in a year?

2007 National

Some data: Antarctica, area $1.5*10^7$ km^2, is covered by a layer of ice 1 mile thick; the density of the ice is 0.9 g/cc. The heat of fusion of ice is 80 cal/g and the heat of condensation of steam is 540 cal/g. The density of sea water is 1.03 g/cc and contains 3 % NaCl. The Earth is a sphere with a diameter of $12.8*10^3$ km. Over its lifetime of 50 years, an average tree will convert 1 ton of CO_2 into cellulose, $(C_6H_{10}O_5)_n$, density 1.4 g/cc. The density of wood from that average tree is 0.6 g/cc. The Earth's dry atmosphere is composed of: (volume %) 78 N_2, 21 O_2, 1 Ar, ~0.04 CO_2. The average water vapor content is ~ 2% by weight. There are 10^8 cattle in the U.S.; each one emits CO_2 at an average rate of 0.5 L/min.

1. How many grams of steam are needed to melt a cubic foot of ice?
2. What is the surface area, in km^2, of the continents of the Earth?
3. What is the weight, in tons, of the Earth's atmosphere?
4. What is the mass, kg, of CO_2 in the Earth's atmosphere?
5. How many grams of oxygen are in the earth's atmosphere?
6. What is the area, cm^2, of the Earth's oceans?
7. If the oceans' average depth is 3000 m, what is their mass, g?
8. How many water molecules provide the mass in problem 7.?
9. What is the ratio of the water in the Earth's atmosphere compared to the amount in the oceans?
10. What is the ratio of the volume of an 8 cm diameter ball to the Earth?
11. If the Earth and its contents were shrunk to an 8 cm ball, how tall, cm, would you be?
12. From problem 11., what is the total height, cm, of the Earth's populace?
13. How high, m, is a column of air, at 0°C, that weighs 14.7 lbs/in^2 ?
14. The barometric pressure can be approximated by the equation $P=A_0*10^{-0.000058*A}$ where A is the altitude, m, above sea level. At what altitude, m, will the pressure drop to ½ the sea level value?

15. What is the air pressure, mm Hg, at 100 km altitude?

16. Estimate the number of gallons of gasoline consumed by automobiles on an average day in the U.S.

17. Assume that gasoline is octane, density 0.7 g/cc. How many tons of carbon dioxide would be created each year by the combustion of gasoline (problem 16.)?

18. What is the ratio of the total CO_2 in the atmosphere to that produced in problem 17. ?

19. How many trees are needed to convert the carbon dioxide produced in a month (problem 17.)?

20. Assuming that the trees are planted 5 m apart, how large an area, km^2, would this forest cover?

21. How much oxygen, tons, will be released each month by the process in problem 19.?

22. What is the mass, g, of the mature average tree?

23. How many grams of CO_2 are emitted per day by cattle in the U.S.?

24. What fraction of the earth's atmosphere is equivalent to the CO_2 produced by cows in a year?

25. What is the ratio of CO_2 produced by cattle to that produced by cars?

26. If 10^5 m^3 of sea water were evaporated, how many grams of salt could be recovered?

27. What fraction of the Earth's water is equivalent to that in the Antarctic ice sheet?

28. What is the ratio of the amount of oxygen in the Antarctic ice sheet to that in the atmosphere?

29. If all of the ice in Antarctica were to melt because of global warming, how high, cm, would the oceans rise?

30. A 1000 ton tanker has a draft of 30 m. If the melted ice of problem 27 floats atop the ocean, how many cm lower will the tanker sink?